**MATHCOUNTS**

**Sprint Round**

**2009**

1. A bookcase has 3 shelves with a total of 24 books. The top 1.

shelf has 8 mystery books. The middle shelf has 10 math

books. The bottom shelf has 6 sience books. Two books

are now taken off each shelf. What fraction of the

books remaining on the shelves are math books?

Express your answer as a common fraction.

2. The Golden Chicken Egg Farm packages eggs in cartons 2.

that hold 12 eggs each. What is the smallest number of

eggs greater than 10,000 that will fill an integer

number of these cartons without any eggs left over?

3. Wilhelm has seven tokens, each with a prime number written 3.

on its top face. He notices that these seven numbers are

distinct, consecutive prime numbers. What is the least sum

of the prime numbers written on Wilhelm’s seven tokens?

4. Let ψ be the relationship defined by A ψ B = 2A +5B. What 4.

is the value of 9 ψ (3 ψ 1)?

5. A flock of geese is swimming on a peaceful lake when a 5.

noisy motorcycle drives past causing $\frac{2}{5}$ of the geese to

fly away. A little later a small herd of deer runs past and

causes $\frac{1}{3}$ of the remaining geese to fly off. Then when a

loud siren sounds, another 24 geese leave the lake. Now

only 46 geese remain on the lake. How many geese were

in the flock to start?

6. The probability it will rain on Saturday is 60%, and the 6.

probability it will rain on Sunday is 25%. If the probability

of rain on a given day is independent of the weather on any

other day, what is the probability that it will rain on both days,

expressed as a percent?

7. Five termites are eating through a piece of wood, 7.

all beginning at the same edge and going in the

same direction. Woody is 20 mm ahead of Muncher,

Cruncher is 10mm behind Woody, Muncher is 5 mm

behind Nibbler, and Biter is 15 mm ahead of Cruncher.

How many millimeters is the distance between the two

 termites that are the farthest apart?

8. Four primes *a, b, c,* and *d* form an increasing arithmetic 8.

sequence with *a* > 5 and a common difference 6. What is

the ones digit of *a*?

9. On planet Larkey, 7 ligs = 4 lags, and 9 lags = 20 lugs. 9.

How many ligs are equivalent to 80 lugs?

10. A tank is to be filled with water. When the tank is 10.

one-sixth full, 130 gallons of water are added, making

tank three-fifths full. How many gallons does the tank

hold when completely full?

11. Thirty students took a test on which it was possible to 11.

earn only integer scores ranging from 3 to 10, inclusive.

If exactly 24 students passed, by earning a score of 7

or higher, what is the highest possible average of the 30

scores? Express your answer as a decimal to the

nearest tenth?

12. The mean of five positive integers is 1.5 times their 12.

median. Four of the integers are 8, 18, 36, and 62,

and the largest integer is not 62. What is the largest

integer?

13. In the diagram shown,  and  13.

If is 3.5 times what is ?

B

C

D

A

O

14. If three standard, six-faced dice are rolled, what is the 14.

probability that the sum of the three numbers rolled is 9?

Express your answer as a common fraction.

15. Alfred, Brandon, and Charles are three participants in 15.

a race. In how many different ways can the three finish

if it is possible for two or more participants to finish in a tie?

16. A rectangular garden has a length that is twice its width. 16.

The dimensions are increased so that the perimeter is

doubled and the new shape is a square with an area of

3600 square feet. What was the original garden, in

square feet?

17. What is the ordered pair (*x*, *y*) where *x* and *y* are integers, 17.

and $2^{x}-2^{y}=8$?

18. Given that the diagonals of a rhombus are always 18.

perpendicular bisectors of each other, what is the area

of a rhombus with side length $\sqrt{89}$ units and the diagonals

differ by 6 units?

19. If the expression $(12-x)÷3x$ represents 19.

a non-negative integer, what is the largest

possible integer value of *x*?

20. Twenty-four 4- inch wide square posts are evenly 20.

spaced with 5 feet between adjacent posts to

enclose a square field, as shown. What is the outer

perimeter, in feet, of the field? Express your answer

as a mixed number.

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21. What is the smallest possible solution of the equation 21.

?

22. The arithmetic progressions and 22.

have some common values.

What is the largest value less than 500 that they

have in common?

23. An isosceles right triangle is removed from each corner 23.

of a square piece of paper, as shown, to create a rectangle.

If units, what is the combined area of the four

A

removed triangles in square units?

B

24. If and , for how many ordered 24.

pairs of integers (*a, b*) is an integer?

25. Six students are being grouped into three pairs to work 25.

on a science lab. How many different combinations

of three pairs are possible?

26. In trapezoid ABCD, is parallel to ,units, 26.

and units. Segment EF is drawn parallel to 

with E lying on and F lying on . If BF:FC =3:4,

What is EF? Express your answer as a common fraction.

27. An integer X has the following properties: 27.

1. X is a multiple of 17
2. X is less than 1000
3. X is one less than a multiple of 8

What is the largest possible value of X?

28. Points X, Y, and Z lie on the sides of 28.

triangle ABC so that segments AX, BY and CZ,

if drawn, would intersect at one interior

point P. Using 3 of these 7 points at a time

as vertices, how many triangles can be formed?

29. The sum of the *n* positive integers, not 29.

necessarily distinct, is 22. What is the

largest possible product of the *n* integers?

30. In the figure, point A is the center of the 30.

circle, the measure of angle RAS is 74 degrees,

and the measure of angle RTB is 28 degrees.

What is the measure of minor arc BR,

in degrees?

R

S

T

B

A