# MATHCOUNTS®

**2011**
■ **State Competition** ■
**Sprint Round**
**Problems 1–30**

Name _____

School _____

Chapter _____

## DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

| Total Correct | Scorer's Initials |
|---|---|
|  |  |
|  |  |

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1. Boy Scout Troop 324 is planning to go hiking for 1 day. There are 12 boys in the troop, and they will be accompanied by 3 scout leaders. Each person will need at least 3 bottles of water that day. What is the minimum number of bottles of water needed for the trip?

1. _____ bottles

2. Cammie has some pennies, nickels, dimes and quarters. What is the least number of coins that she can use to make 93 cents?

2. _____ coins

3. Kara has three non-overlapping circles. The area of the circle 8 inches in diameter is how much greater than the combined areas of the circle 6 inches in diameter and the circle 2 inches in diameter? Express your answer in terms of $\pi$.

3. _____ sq in

4. During the first five games of the baseball season, Mike Slugger had 7 hits in 20 turns at bat. In the next game, he had 5 hits in 5 turns at bat. For the first six games, what percent of Mike's turns at bat resulted in hits?
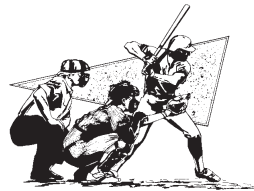
4. _____ %

5. The first term of an arithmetic sequence is –37, and the 2nd term is –30. What is the smallest positive term of the sequence?

5. _____

6. At Paul Bunyan's tree farm they sell Fraser firs and blue spruce trees. They plant saplings of these two kinds of trees that are 8 inches and 5 inches tall, respectively. The Fraser firs grow at a constant rate of 12 inches per year, and the blue spruce trees grow at a constant rate of 14 inches per year. After how many years will these trees be the same height? Express your answer as a common fraction.

6. _____ years

7. I had $30 in allowance money and spent it as indicated in the pie graph shown. How many dollars did I spend on burgers?

**How I Spent My Allowance**

Movies $\frac{1}{3}$

Music $\frac{3}{10}$

Burgers

Ice Cream $\frac{1}{5}$
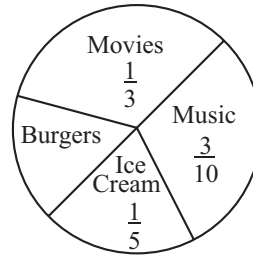
7. $ _____

8. M and N are both perfect squares less than 100. If M − N = 27, what is the value of $\sqrt{M} + \sqrt{N}$?

8. _____

9. John made two 120-mile trips. He made his second trip in one hour less time than his first trip. The total time for the two trips was 9 hours. What was his average rate, in miles per hour, for the second trip?

9. _____ mph

10. In the figure shown, there are parallelograms of many sizes. How many total parallelograms are there in the diagram?
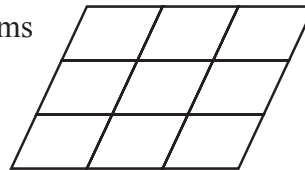
10. _____ parallelograms

11. Five centimeters more than three times the length of a rectangle is less than or equal to 44 cm and greater than or equal to 20 cm. The width of the same rectangle is 10 cm. What is the positive difference between the maximum possible area of the rectangle and the minimum possible area of the rectangle?

11. _____ sq cm

12. Javier needs to exchange his dollar bill for coins. The cashier has 2 quarters, 10 dimes and 10 nickels. Assuming the cashier gives Javier the correct amount and at least one quarter, how many possible combinations of coins could Javier receive?

12. _____ combinations

13. Joan has four test scores. The median is 80 points, and the range is 12 points. What is the maximum possible score she could have received on a test?

13. _____ points

14. The five-digit integer RS,TUV is divisible by 5, $S = R^2$ and $10 \times T + U = 5$. If R, S, T, U and V are not necessarily distinct, how many positive five-digit integers satisfy these conditions?

14. _____ integers

15. Given the function $y = x^2 + 10x + 21$, what is the least possible value of $y$?

15. _____

16. A square and a circle intersect so that each side of the square contains a chord of the circle equal in length to the radius of the circle. What is the ratio of the area of the square to the area of the circle? Express your answer as a common fraction in terms of $\pi$.
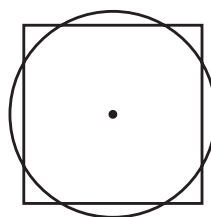
16. _____

17. In a rectangle ABCD, point E is on side CD. The area of triangle ADE is one-fifth of the area of quadrilateral ABCE. What is the ratio of the length of segment DE to the length of segment DC? Express your answer as a common fraction.

17. _____

18. For a certain sequence of numbers, the sum of the first $n$ numbers in the sequence is given by $n^3 + 4n$ for all positive integers $n$. What is the tenth number in the sequence?

18. _____

19. The figure shows a square in the interior of a regular hexagon and sharing a common side. What is the degree measure of ∠ABC?

A

C

B
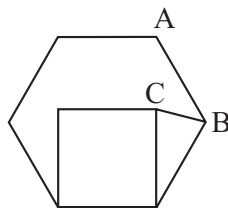
19. _____ degrees

20. When the diameter of a pizza increases by 2 inches, the area increases by 44%. What was the area, in square inches, of the original pizza? Express your answer in terms of $\pi$.

20. _____ sq in

21. If the numerator of a fraction is increased by six, the value of the fraction will increase by one. If the denominator of the original fraction is increased by 36, the value of the original fraction will decrease by one. What is the original fraction? Express your answer as a common fraction.

21. _____

22. The 80th term of an arithmetic sequence is twice the 30th term. If the first term of the sequence is 7, what is the 40th term?

22. _____

23. A right circular cone is sliced into four pieces by planes parallel to its base, as shown in the figure. All of these pieces have the same height. What is the ratio of the volume of the second-largest piece to the volume of the largest piece? Express your answer as a common fraction.
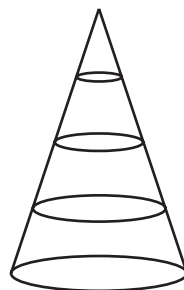
23. _____

24. If $x + y + z = 7$ and $x^2 + y^2 + z^2 = 19$, what is the arithmetic mean of the three products $xy$, $yz$ and $xz$?

24. _____

25. Quadrilateral ABCD is inscribed in a circle with segment AC a diameter of the circle. If $m\angle DAC = 30°$ and $m\angle BAC = 45°$, the ratio of the area of ABCD to the area of the circle can be expressed as a common fraction in simplest radical form in terms of $\pi$ as $\dfrac{a+\sqrt{b}}{c\pi}$. What is the value of $a + b + c$?

25. _____

26. The numbers $a$, $b$, $c$ and $d$ form a geometric sequence, in that order. If $b$ is three more than $a$, and $c$ is nine more than $b$, what is the value of $d$? Express your answer as a mixed number.

26. _____

27. In three flips of an unfair coin the probability of getting three heads is the same as the probability of getting exactly two tails. What is the ratio of the probability of flipping a tail to the probability of flipping a head? Express your answer as a common fraction in simplest radical form.

27. _____

28. The region shown is bounded by the arcs of circles having radius 4 units, having a central angle measure of 60 degrees and intersecting at points of tangency. The area of the region can be expressed in the form $a\sqrt{b} + c\pi$ square units, where $\sqrt{b}$ is a radical in simplest form. What is the value of $a + b + c$?

28. _____
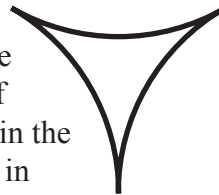
29. A bag contains red balls and white balls. If five balls are to be pulled from the bag, with replacement, the probability of getting exactly three red balls is 32 times the probability of getting exactly one red ball. What percent of the balls originally in the bag are red?

29. _____ %

30. What is the value of $52{,}683 \times 52{,}683 - 52{,}660 \times 52{,}706$?

30. _____