**MATHCOUNTS**

**State Competition**

**Sprint Round 1993-1994**

1. In the diagram both circles have center O,  1.

and are diameters of the larger circle. ,

AO = 2, and EO = 1. Find the area of the shaded region.

Express your answer as a common fraction in terms of π.

B

A O E C

D

2. A fair 8-sided die has faces numbered 2.

1, 2, 3, 4, 5, 6, 7, and 8. If the die is rolled twice, what is

the probability that the sum of the numbers will be 9?

Express your answer as a common fraction.

1. A single large banquet table is created by joining the 3.

sides of several small square tables, each of which seats

one person for each open side. What is the maximum

number of people that could be seated at a table which

uses 10 smaller tables?

4. An 8 inch by 10 inch rectangular picture is surrounded 4.

by a border that is 2 inches wide on all sides. Find the ratio

of the area of the border to the area of the picture.

Express your answer as a common fraction.

5. If and , what is the 5.

value of x + y? Express your answer as a

common fraction.

6. Every member of a math club is taking algebra 6.

or geometry and 8 are taking both. If there are

17 taking algebra and 13 taking geometry,

how many members are in the club?

7. Two factors have a product of 3·92x - 6·27x. 7.

One of the factors is 3x – 2. What is the other

factor?

8. A regular hexagon of side 7 is surrounded 8.

on each side by regular hexagons of the same size

adjacent to its sides. What is the perimeter

of the resulting figure?

9. If 2n is a factor of 20!, what is the largest 9.

possible value of n?

10. A wrecker’s iron ball eight inches in diameter 10.

weighs 120 pounds. How many pounds would

a similar iron wrecking ball twelve inches

in diameter weigh?

11. A rectangle XYZW has XY = 6 and XW = 8. 11.

If a point M is selected randomly in the interior of

this rectangle, what is the probability that triangle

XMW has an area equal to or greater than

16 square units? Express your answer as a

common fraction.

12. Fruit salad can be made with any 3 of these 12.

5 fruits: apples, bananas, grapes, strawberries,

and pineapples. If strawberries and pineapples

do not taste good together and grapes and bananas

do not look appetizing together, how many possible

good tasting and appetizing salads are there?

13. A group of boys and girls reserved a single 13.

row of 120 seats at a theater. The girls were

assigned seat numbers in such a way that every boy

had to sit next to at least one girl. What is the fewest

number of girls necessary to make this happen?

14. A and B are non-zero digits for which A468B05 14.

is divisible by 11. What is A + B?

15. Point Z is the midpoint of MN. Point P is the midpoint 15.

of ZN. Point Q is the midpoint of PN and point R is the

midpoint of QN. If PR is 24 inches long, how many feet

long is MN? Express your answer as a mixed number.

16. To the nearest hundredth of a gallon, how many 16.

gallons of gasoline can be saved each year by driving a

car that gets 32 miles per gallon as opposed to a car that

gets 18 miles per gallon. Assume the number of miles

driven each year is 9,000.

17. Find the value of K for which these two equations 17.

will not have a common solution:

6x + 4y = 7

Kx + 8y = 7

18. Which of the fractions is the smallest? 18.

13/40, 17/52, 21/64, 25/76?

19. Massaru has 3 pencils in his book bag. Two pencils 19.

are blue and one is red. If he randomly selects two

pencils, what is the probability that they are the same color?

Express your answer as a common fraction.

20. The hour hand of a broken clock points to the 5 20.

and the minute hand points to the 12. To the nearest

percent what part of the clock face is shaded between

the hour and minute hand?

21. Suppose A and B are non-zero digits and A5·B1 = 1995. 21.

Find A + B.

22. A sequence of integers is obtained by starting with three 22.

digits d1, d2, and d3. The fourth term of the sequence is

the unit's digit of d1+ d2 + d3. Each succeeding term is the

unit’s digit of the sum of the three previous terms. What

numbers belong in the first, second, and third positions

respectively to complete the sequence?

\_\_\_, \_\_\_, \_\_\_, 1, 1, 1

23. If A ◊ B = , what is the value of 23.

(3 ◊ 4) ◊ 5? Express your answer as a common fraction.

23.

24. What integer is closest to  24.

25. How many positive numbers have cube roots 25.

less than 1 + ?

26. Find the sum of . 26.

Express your answer as a common fraction. 26.

27. The sum of the digits is of a two-digit number 27.

is 12. When the digits are reversed, the original

number is increased by 54. What is the product

of the digits?

28. Out of 22 students surveyed on ice cream 28.

flavors, 12 liked chocolate, 5 liked strawberry,

and 6 liked vanilla. If 3 liked chocolate and vanilla,

how many students did not like any of these flavors?

29. What is the value of x in 29.

222x - 111x = 111x · 7?

30. There exist non-negative integers a and b 30.

such that for any x satisfying a < x < b, it is true that

. Find the ordered pair (a, b).