**MATHCOUNTS**

**Sprint Round**

**2007**

1. In the figure to the right, the largest circle 1.

has a radius of six meters. Five congruent

smaller circles are placed as shown and

are lined up in east-to-west and north-to-south

orientations. What is the radius of one

of the five smaller circles?

2. Two identical CDs regularly cost a total of $28. 2.

What is the cost of five of these CDs?

3. Alexandra played 15 games of Skee Ball for $11.25. 3.

After each game, she earned either 2 tickets or 4 tickets.

At the end of 15 games she had earned a total of 40 tickets.

For how many games did she earn 4 tickets?

4. In parallelogram PQRS, the measure of angle P is 4.

five times the measure of angle Q. What is the measure

of angle R, in degrees?

5. Two right triangles have equal areas. The first triangle 5.

has a height of 5 cm and a corresponding base of 8 cm.

The second triangle has a leg of length 20 cm.

What is the length of the other leg of the second triangle?

6. In a class of 30 students, 12 of the students are girls. 6.

If six more boys were to join the class, what fraction

of the class would be boys?

Express your answer as a common fraction.

7. The distance from the earth to the moon is 7.

approximately miles. A satellite

crosses the shortest path between the earth

and the moon at a distance of 240 miles from

the earth. What fraction of the distance from

the earth to the moon is the satellite’s distance

from the earth at this point? Express your answer

as a common fraction with the numerator and

denominator *each in integer form*.

8. How many perfect squares less than 1000 have 8.

a ones digit of 2, 3 or 4?

9. Over time, a patient needs to get allergy shots 9.

totaling 60 mL. If the first 20 shots are each 0.5 mL,

the next 20 shots are each 1 mL and thereafter each

shot is 2 mL, how many shots does the patient get altogether?

10. Louisa ran at an average speed of five miles per hour 10.

along an entire circular park path. Calvin ran along the

same path in the opposite direction at an average speed

of six miles per hour. It took Calvin 30 minutes less than

it took Louisa to run the full path once. How many miles

did Louisa run when she completed one circular path?

11. The cost of renting a bus for a field trip was split evenly 11.

among 20 students. At the last minute, 10 more students

joined the trip. The cost of renting the bus was then evenly

redistributed among all of the students. The cost for each

of the original 20 students decreased by $1.50.

What was the total cost of renting the bus?

12. If the date is March 1, 2007, in how many whole 12.

years will the number representing the year be the

smallest perfect square greater than 2007?

13. In this quilt pattern, points E, F, G and H are 13.

midpoints of the sides of square ABCD, and

square EFGH is divided into nine congruent A E B

unit squares, as shown. What percent of the

total area of square ABCD does the total

shaded area represent? Express your answer H F

to the nearest whole percent.

 D G C

14. Twenty people are seated around a circular table. 14.

Each person is either a knight, who always tells the

truth, or a knave, who always lies. If each person

at the table says, “The person on my right is a knave,”

how many knaves are seated around the table?

15. At an immunization clinic, there is one scheduled 15.

appointment every five minutes starting at noon.

Each patient arrives exactly at the scheduled time,

spends 10 minutes in the lobby doing paperwork,

then five minutes out of the lobby getting shots, and

finally 20 minutes in the lobby to be sure there is

no adverse reaction. How many patients are in

the lobby at 12:34 p.m.?

16. A data set for a class of 25 sixth graders has 16.

their ages listed as the integer values of either

10 or 11 years. The median age in the

data set is 0.36 years greater than the mean.

How many 10-year-olds are in the class?

17. What is the greatest number of whole 4-inch 17.

by 6-inch notecards that can be cut from a

25-inch by 27-inch sheet of poster board if

every cut must be made parallel to a side of

the poster board? No taping or gluing is allowed.

18. Brand X soda advertises, “We will give you 20% 18.

more soda than Brand Y for a total price that is

10% less than Brand Y’s price!” What is the ratio

of the unit price of Brand X soda to the unit price

of Brand Y soda?

Express your answer as a common fraction.

19. A “size 2 square of squares” is a 2 by 2 grid of digits 19.

where the digits in each row, from left to right,

form a two-digit perfect square, and the digits in

each column, from top to bottom, also form a two-digit

perfect square.

How many “size 2 squares of squares” are there?

20. A sheet of notebook paper weighs 0.005 ounces 20.

per square inch. A box contains 24 notebooks.

Each notebook has 100 sheets of 10-inch by 6-inch

paper. For each notebook, the back cover and

the front cover together weigh one-half ounce.

What is the total weight of the notebooks in the box?

21. The product of two positive fractions is $\frac{1}{9}$. 21.

The larger fraction divided by the smaller fraction is 4.

What is the sum of the two fractions?

Express your answer as a common fraction.

22. When five standard six-sided dice are rolled sequentially 22.

there are = 7776 possible outcomes. For how many

outcomes is the sum of the five rolled numbers exactly 27?

23. A box contains some green marbles and exactly 23.

four red marbles. The probability of selecting a

red marble is *x*%. If the number of green marbles is

doubled, the probability of selecting one of the

four red marbles from the box is (*x* - 15)%.

How many green marbles are in the box before

the number of green marbles is doubled?

24. Julie baked cupcakes for her family at home and 24.

for a party at school. She iced 4 cupcakes with red frosting,

2 cupcakes with orange frosting, 2 with yellow,

2 with green, 3 with blue and the last 3 with violet frosting.

Each cupcake is iced with exactly one color of frosting.

Julie plans to take exactly 10 of the cupcakes

to her party, and will take either all of the cupcakes of

a particular color or none of the cupcakes of that color.

How many different combinations of cupcakes could

she take to her party?

25. Monica reduced  to a common fraction by canceling 25.

the middle 5s to get  . This answer is correct, but the

procedure is incorrect. Kia’s example worked with

Monica’s incorrect procedure, too; the fraction

also reduced to its equivalent common fraction when

the identical middle digits of the two three-digit integers

were canceled out. What is the greatest possible value of *x*?

26. The area of the semicircle in Figure A is half 26.

the area of the circle in Figure B. The area of a square

inscribed in the semicircle, as shown, is what fraction

of the area of a square inscribed in the circle?

Express your answer as a common fraction.

 *Figure A Figure B*

27. A collection of 20 coins made up of only nickels, 27.

dimes and quarters has a total value of $3.35.

If the dimes were nickels, the nickels were quarters

and the quarters were dimes, the collection of coins

would have a total value of $2.75.

How many quarters are in the original collection?

28. Firefighter Steve was born in 1950. Steve had a 28.

grandmother who was born in a year which is the

product of two prime numbers, one of which is one less

than twice the other. In what year did Steve’s

grandmother celebrate her 90th birthday?

29. The digits 2, 3, 4, 7 and 8 will be put in random 29.

order to make a positive five-digit integer.

What is the probability that the resulting integer will

be divisible by 11?

Express your answer as a common fraction.

30. Right triangle ABC has one leg of length 6 cm, 30.

one leg of length 8 cm and a right angle at A.

A square has one side on the hypotenuse of triangle ABC

and a vertex on each of the two legs of triangle ABC. What is

the length of one side of the square?

Express your answer as a common fraction.

 A

 B C