**MATHCOUNTS**

**Sprint Round**

**2010**

1. Sharon bought a mixture of nuts that was made up of 1.

pecans, walnuts, and cashews in a ratio of 3:2:1

respectively. If she bought 9 pounds of nuts, how many

pounds of walnuts were in the mixture? Express your

answer as a decimal to the nearest tenth.

2. A bookcase has 3 shelves. On the top shelf there are 2.

10 German books, on the middle shelf there are 12 math

books and on the bottom shelf there are 8 science books.

When six of the German books are removed from the

bookcase, what fraction of the books are math? Express your

answer as a common fraction.

3. What is the sum of the units digits of all multiples of 3 3.

between 0 and 50?

4. An item in a store is originally priced at $4.00. A clerk 4.

marks up the item by 100%. After that, a second clerk

marks the new price up by 200%. What was the final

price after the two markups?

5. The points and are two non-consecutive 5.

vertices of a rhombus with an area of 80 square units.

One of the other vertices is , where *K* > 0.

What is the value of *K*?

6. Jan is as old as Gary was 15 years ago. Six years from now 6.

Gary will be twice as old as will be then. How old is Gary now?

7. If three flick are equivalent to eight flecks and six flocks are 7.

equivalent to four flecks, how many flocks are

equivalent to 12 flicks?

8. The integer *m* is between 30 and 80 and is a multiple 8.

of 6. When *m* is divided by 8, the remainder is 2.

Similarly, when *m* is divided by 5, the remainder is 2.

What is the value of *m*?

9. It took 4 days for 75 workers, all working at the same rate, 9.

to build an embankment. If only 50 workers had been

available, how many total days would it have taken to

build the embankment?

10. Hector spent a total of exactly $200 on shirts and pants 10.

The shirts cost $15 each and the pants cost $22 each.

How many shirts did Hector buy?

11. How many ounces of pure water must be added to 11.

30 ounces of a 30% solution of acid to make it a

20% solution?

12. If only triangles may be used, how many additional 12.

triangles must be added to the right side of the third

scale so that it is balanced? (The distance of these

objects from the centers of theses scales is not relevant.)

13. The sum of the first 20 positive even integers is also the 13.

sum of four consecutive even integers. What is the

largest of these four integers?

14. Six black chips have been placed 14.

3

1



on the game board shown. What is

the sum of the numbers in the two

7

6

squares where the seventh and eighth

8

12

11

chips must be placed so that each row

contains exactly 2 chips and each

14

13

16

column contains exactly 2 chips?

15. A set of seven distinct positive integers has its mean and its 15.

median both equal to 30. What is the largest possible integer

this set can contain?

16. Three coplanar squares with sides of lengths two, four and 16.

six units, respectively, are arranged side-by-side, as shown

so that one side of each square lies on line AB and a segment

connects the bottom left corner of the smallest square to the

upper right corner of the largest

square. What is the area of the

6

shaded quadrilateral?

4

2

B

A

17. The fifth term of a geometric sequence of positive numbers 17.

is 11 and the eleventh term is 5. What is the eighth term of

the sequence? Express your answer in simplest radical form.

**11**

**5**

18. The vertices of a convex 18.

pentagon are (−1, −1), (−3, 4),

(1, 7), (6, 5) and (3, −1).

What is the area of the pentagon?

19. A car passes point A driving at a constant rate of 60 km 19.

per hour. A second car, traveling at a constant rate of

75 km per hour, passes the same point A a while later and

 then follows the first car. It catches the first car after

traveling a distance of 75 km past point A. How many

minutes after the first car passed point A did the second

car pass point A?

20. When the square of three times a positive integer is 20.

decreased by the integer, the result is 2010.

What is the integer?

21. Bag A contains 3 white and 2 red balls. Bag B contains 21.

6 white and 3 red balls. One of the two bags will be chosen

at random, and then two balls will be drawn from that bag

at random without replacement. What is the probability that

 the two balls drawn will be the same color?

Express your answer as a common fraction.

22. The base 9 representation of a positive integer is AB 22.

and its base 7 representation is BA. What is the integer

expressed in base 10?

23. For how many positive values of *n* are both 3*n* and 3*n* four- 23.

digit integers?

24. If 3x + y = 81 and 81x – y = 3, then what is the value of the 24.

product *xy*? Express your answer as a common fraction.

25. In rectangle ABCD, points E and F lie on segments AB 25.

and CD, respectively, such that AE = $\frac{AB}{2}$ and CF = $\frac{CD}{2}$.

Segment BD intersects segment EF at P. What fraction

of the area of rectangle ABCD lies in triangle EBP?

Express your answer as a common fraction.

26. The points (1, 7), (13, 16) and (5, *k*), where *k* is an integer, 26.

are vertices of a triangle. What is the sum of the values of *k*

for which the area of the triangle is a minimum?

27. What is the value of *x* for which 27.

 $\sqrt{x+\sqrt{x+\sqrt{x+…}}}=5$

28. If *a* and *b* are integers with *a* > *b*, what is the smallest 28.

possible positive value of $\frac{a+b}{a-b}+\frac{a-b}{a+b}?$

29. The positive integers A, B and C form an arithmetic 29.

 sequence while the integers B, C and D form a geometric

sequence. If $\frac{C}{B}=\frac{5}{3}$, what is the smallest possible value

of A + B + C + D?

30. Point D lies on side AC of equilateral triangle 30.

ABC such that the measure of angle DBC is 45

degrees. What is the ratio of the area of triangle

ADB to the area of triangle CDB? Express your p

answer as a common fraction in simplest radical form.