**MATHCOUNTS**

**Team Round**

**2010**

1. What is the largest five-digit integer whose digits 1.

have a product equal to the product (7)(6)(5)(4)(3)(2)(1)?

2. In triangle ABC, the measure of ∠A is 86 degrees. The 2.

measure of ∠B is 22 degrees more than three times the

measure of ∠C. What is the measure, in degrees, of ∠C?

3. Given that a particular positive integer is a four-digit 3.

palindrome, what is the probability that it is a multiple

of 99? Express your answer as a common fraction.

4. What is the largest integer less than 2010 that has 4.

a remainder of 5 when divided by 7, a remainder of 10

when divided by 11, and a remainder of 10 when divided by 13?

5. The measures of the interior angles of a convex 5.

hexagon form an increasing arithmetic sequence.

How many such sequences are possible if the hexagon is

not equiangular and all of the angle degree measures are

positive integers less than 150 degrees?

6. Six-digit integers will be written using each of the digits 6.

1 through 6 exactly once per six-digit integer. How many

different positive integers can be written such that all pairs

of consecutive digits of each integer are relatively prime?

(Note: 1 is relatively prime to all integers.)

7. A sheet of paper 12 inches by 18 inches is folded so 7.

that two opposite corners touch, as shown in the figures

 below. What is the area, in square inches, of the shaded

B

triangle formed where the paper overlaps itself?

A, C

B

A

C

D

D

8. A rectangular box has a volume of 4320 cubic inches 8.

and a surface area of 1704 square inches. The sum of the

lengths of its 12 edges is 208 inches. What would be the

volume of the box, in cubic inches, if its length, width

and height were each increased by one inch?

9. Two numbers between 0 and 1 on a number line are to be 9.

chosen at random. What is the probability that the second

 number chosen will exceed the first number chosen by a

 distance greater than $\frac{1}{4}$ unit on the number line?

Express your answer as a common fraction.

10. A square and isosceles triangle of equal height are 10.

side-by-side, as shown, with both bases on the *x*-axis.

The lower right vertex of the square and the lower left

vertex of the triangle are at (10, 0). The side of the square

 and the base of the triangle on the *x*-axis each equal 10 units.

A segment is drawn from the top left vertex of the square

to the farthest vertex of the triangle, as shown.

What is the area of the shaded region?

*y*

*x*

(10, 0)